C.U.SHAH UNIVERSITY

WADHWAN CITY

Summer Examination-May 2015

Course Name: M.Sc-II Subject Name : Real analysis-I(5SC02MTC4) Marks: 70

Duration : 3 Hours

Instructions:

1) Attempt all Question of both sections in same answer book/supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION - I

Q-1 (A) Define measurable set. [01]

(B) Prove that a countable set has measure zero. [02]

(C) If any set of outer measure zero, then show that it is measurable. [02]

(D) Prove that the translate of a measurable set is measurable. [02]

Q-2 (A) Show that Lebesgue measure is countably additive. [07]

(B) Prove that any set of real numbers with positive outer measure contains a subset that fails to be measurable. [07]

OR

Q-2 (A) Let f and g are measurable function on that are finite a.e. on E. Then prove that, for any α and β, $(\alpha f + \beta g)$ and (fg) are measurable on E. [07]

(B) Let E be a set with finite measure and $\{f_n\}$ be a sequence of measurable functions on E that converges point-wise on E to the real-valued function f. Then for each $\varepsilon > 0$, there is a closed set F contained in E for which $\{f_n\} \to f$ uniformly on F and $m(E \sim F) < \varepsilon$. [07]

Q-3 (A) If $\{A_k\}_{k=1}^{\infty}$ and $\{B_k\}_{k=1}^{\infty}$ are ascending and descending collection of measurable sets respectively and $m(B_1) < \infty$, then prove that $m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(A_k)$ and $m\left(\bigcap_{k=1}^{\infty} B_k\right) = \lim_{k\to\infty} m(B_k)$ [07]

(B) Let E be a measurable set of finite outer measure. Then for each $\varepsilon > 0$, there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^{n}$ for which if $\mathrm{O} = \bigcup_{k=1}^{n} I_k$, then $m^*(E \sim \mathrm{O}) + m^*(\mathrm{O} \sim E) < \varepsilon$. [07]

OR

Q-3 (A) Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise *a.e.* on E to the function f. Then f is measurable. [07]

(B) Let the function f have a measurable domain E. Then the following statement are equivalent: [07]

(a) For each real number c, the set $\{x \in E \mid f(x) > c\}$.

(b) For each real number c, the set $\{x \in E \mid f(x) < c\}$.

SECTION - II

Q-4 (A) Define Lebesgue integrable function. [01]

(B) Give an example of function which is Lebesgue integrable but not Riemann integrable. [02]

(C) Let f be a measurable function on E, then f^+ and f^- are integrable over E if and only if $|f|$ is integrable over E. [02]

(D) Let f be a nonnegative measurable function on E. Then $\int_E f = 0$ if and only if $f = 0$ $a.e.$ on E. [02]

Q-5 (A) State and prove the Vitali converging lemma. [07]

(B) Let f be a bounded function on the closed, bounded interval [a, b]. Then f is Riemann integrable [a, b] over if and only if the set of points in [a, b] at which f fails to be continuous has measure zero. [07]

OR

Q-5 (A) Let f and g be bounded measurable functions on a set of finite measure E. Then for any α and β, $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$ and if $f \le g$ on E, then $\int_E f \le \int_E g$. [07]

(B) If the function f is monotone on the open interval (a,b), then it is differentiable almost everywhere on (a,b). [07]

Q-6 (A) State and prove the Chebychev's inequality. [07]

(B) State and prove the Lebesgue dominated convergence theorem. [07]

OR

Q-6 (A) Let f be a measurable function on E. Suppose there is a nonnegative function g that is integrable over E and dominates f in the sense that $|f| \le g$ on E. Then f is integrable over E and $\left|\int_E f\right| \le \int_E |f|$. [07]

(B) Let $\{f_n\}$ be a sequence of nonnegative integrable function on E. Then $\lim_{n\to\infty} \int_E f_n = 0$ if and only if $\{f_n\} \to 0$ in measure on E and $\{f_n\}$ is uniformly integrable and tight over E. [07]